Large Kerr susceptibility at slow light level via EIT in quantum wells

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Abstract: Large Kerr susceptibilities are investigated in a three level semiconductor multiple quantum well with ladder-type excitation scheme in the regime of electromagnetically induced transparency. We report to have identified the existence of large Kerr (or third order) nonlinear susceptibility of the order $\sim 10^{-12}\,\mathrm{m^2/V^2}$ at pump wavelength of $\sim 6.7\,\mu m$. The group velocity of the probe pulse also has been reported to be slowed down by 10^4 times compared to the speed of light in vacuum.

***Key words**:* Kerr nonlinearities, electromagnetically induced transparency, Rabi frequency, density matrix formalism

Introduction

In recent years, tremendous research effort has been exerted both theoretically as well as experimentally towards the enhancement of optical susceptibilities and to reduce the of group velocity by using electromagnetically induced transparency (EIT) in semiconductor quantum wells (SQWs) [1-3]. This keen interest is due to their potential applications in nonlinear optics with low power level. It is well known that EIT can be used to retardate a light pulse in a dispersive media, sometimes even bring them to a complete stop [2,3]. This excellent property has attracted great attention due to the potential applications in optoelectronics and solid state quantum information science [4-6]. The devices based on intersubband transition (ISBT) in SQW have inherent advantages over those atomic systems, such as large dipole moments due to the small effective electron mass, great flexibility in device design by choosing the materials and structure dimensions, high nonlinear optical coefficients, transition energies as well as symmetries that can also be engineered as desired [7,8]. The devices based on intersubband transition in SQW have great flexibility in device design by choosing the materials and structure dimensions [9]. In the present paper we have investigated the response of a probe laser pulse in a cascade type three level semiconductor multiple quantum well (MQW) system driven by an additional coupling field. We have studied the EIT phenomenon by employing density matrix method and obtain the Kerr nonlinear susceptibilities at the probe laser frequency.

Physical Model and Governing Equations

We have considered a three level semiconductor quantum well structure with ladder-type excitation configuration as shown in Fig.1 (a) [10], where all possible transitions are dipole allowed. A weak probe laser pulse with angular frequency ω_p is coupled to to $|1\rangle \rightarrow |2\rangle$ transition and a strong control laser beam with angular frequency ω_c is coupled to $|2\rangle \rightarrow |3\rangle$ transition, as shown in Fig. 1(a). The system considered here can be designed to possess transition energies $E_{21} = 128\,meV$ and $E_{32} = 182\,meV$ with matrix elements $z_{21} = 1.87\,nm$ and $z_{32} = 2.76\,nm$ respectively. The semi-classical Hamiltonian of the system in the Schrödinger picture is given by,

$$\hat{H} = \sum_{i=1}^{3} \hbar\omega_i |i\rangle\langle i| - \hbar\left\{\Omega_p e^{i(k_p.z-\omega_p t)}|2\rangle\langle 1| + \Omega_c e^{i(k_c.z-\omega_c t)}|3\rangle\langle 2| + h.c.\right\} \quad (1)$$

where $\hbar\omega_i$ is the energy associated with the state $|i\rangle$ of the QW structure and $|i\rangle\langle j|$ are projection operators. Ω_p and Ω_c are the Rabi frequencies for the laser driven intersubband transitions $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|3\rangle$, respectively, which are defined as $\Omega_p = \frac{(\hat{\mu}_{13}.\hat{e}_p)E_p}{\hbar}$ and $\Omega_c = \frac{(\hat{\mu}_{23}.\hat{e}_c)E_c}{\hbar}$. The $\hat{\mu}_{mn} =$

$e\langle m|z|n\rangle$ is the dipole matrix elements for the transition $|m\rangle \leftrightarrows |n\rangle$. The density-matrix equations of motion for the system can be written as follows,

$$\dot{\rho}_{21} = i\left(\Delta_p + i\gamma_{21}\right)\rho_{21} + i\Omega_p(\rho_{11} - \rho_{22}) + i\Omega_c^*\rho_{31} \quad (2)$$

$$\dot{\rho}_{32} = i(\Delta_c + i\gamma_{32})\rho_{32} + i\Omega_c(\rho_{22} - \rho_{33}) + i\Omega_p^*\rho_{31} \quad (3)$$

$$\dot{\rho}_{31} = i\left(\Delta_p + \Delta_c + i\gamma_{31}\right)\rho_{31} + i\Omega_c\rho_{21} - i\Omega_p^*\rho_{32} \quad (4)$$

where intersubband transition detunings Δ_p and Δ_c are defined as $\Delta_p = \omega_p - \frac{(E_2 - E_1)}{\hbar}$, $\Delta_c = \omega_c - \frac{(E_3 - E_2)}{\hbar}$ and γ_{31}, γ_{32}, γ_{21} are the population decay rates. The susceptibility can be divided into linear and nonlinear parts as follows: $\chi_P = \chi^{(1)}(\omega_p) + \chi^{(3)}|E_P|^2$, where we have retained only up to third order terms and neglected higher order terms. The linear and the third order susceptibilities at the probe frequency can be written as follows:

$$\chi^{(1)} = \frac{N|\mu_{13}|^2}{\hbar\varepsilon_0\Omega_p}\rho_{21}^{(1)}, \quad (5)$$

$$\chi^{(3)} = -\frac{N|\mu_{13}|^2}{\hbar\varepsilon_0\Omega_p|E_P|^2}\left(\left|\rho_{21}^{(1)}\right|^2 + \left|\rho_{31}^{(1)}\right|^2\right)\rho_{21}^{(1)}, \quad (6)$$

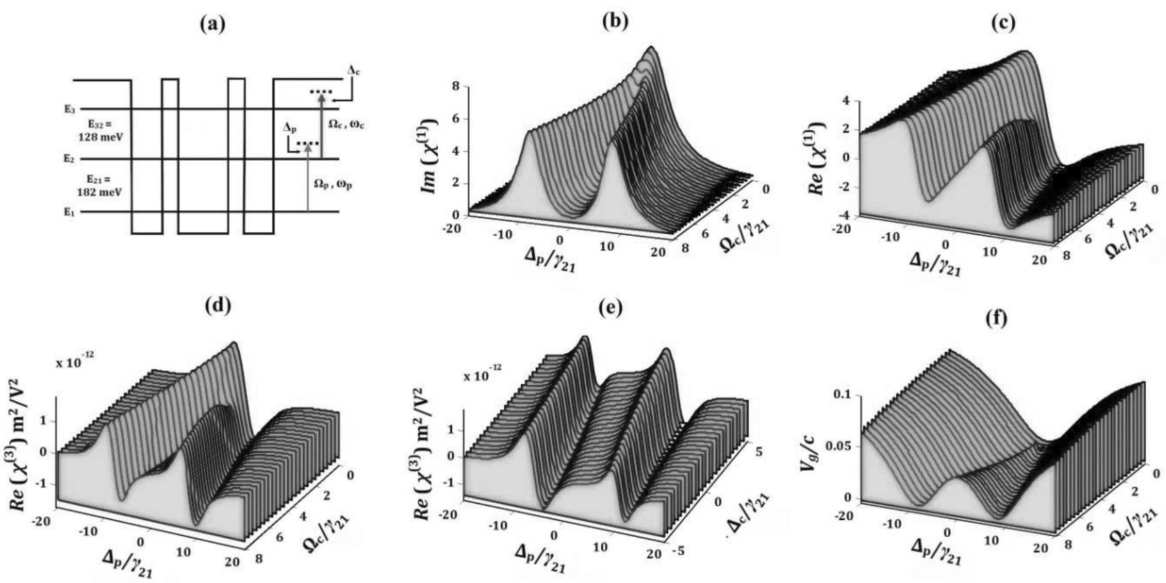

Fig-1: *(a) Excitation scheme for the QW under consideration. Waterfall of imaginary (b) and real (c) parts of linear susceptibilities with the variation of control field Rabi frequency. (d) Real parts of Kerr susceptibility with the variation of control field Rabi frequency, (e) real parts of Kerr susceptibility with the variation of control field detuning and (f) normalized group velocity as functions of normalized probe detuning for different values of control field Rabi frequency. Other system parameters are mentioned in the text.*

The density equations (2) to (3) leads to solutions $\rho_{21}^{(1)}$ and $\rho_{31}^{(1)}$ of the equations (5) and (6). We are now in a position to analyse the first and the third order nonlinear susceptibilities taking into account different parameters associated with the MQW system.

Results and Discussions

Though our primary concern is to investigate the third order or Kerr nonlinear susceptibilities, but for sake of understanding the EIT phenomenon, we first discuss the first order or linear susceptibilities. For our investigation, the system parameters taken as, $N = 10^{22}\, m^{-3}$, $\mu_{12} = 29.90 \times 10^{-29}\, mC$, $\omega_p = 27.65 \times 10^{13} s^{-1}$, decay rates $\gamma_{21} = 10^{12} s^{-1}$ and $\gamma_{31} = 4.10 \times 10^{12} s^{-1}$. The imaginary part of the first order susceptibility $Im\left(\chi^{(1)}\right)$ represents the linear absorption while the real part $Re\left(\chi^{(1)}\right)$ has a direct relation to the linear refractive index of the system. From the profile of $Im\left(\chi^{(1)}\right)$ in Fig. 1(b), we have seen that when the control field is switched off i.e. $\Omega_c/\gamma_{21} = 0$, the probe field at the central frequency ($\Delta_p/\gamma_{21} = 0$) is largely absorbed appearing a strong absorption peak. With the enhancement of Ω_c/γ_{21}, the absorption profile ($Im\left(\chi^{(1)}\right)$) splits into two separate peaks, which is the signature of forming the EIT window. Meanwhile, $Re\left(\chi^{(1)}\right)$ changes from positive to negative at $\Omega_c/\gamma_{21} = 0$ as shown in Fig. 1 (c). With the enhancement of Ω_c/γ_{21}, within the TW the profile of $Re\left(\chi^{(1)}\right)$ reverses, indicating the change in group velocity from anomalous dispersion region to normal dispersion region.

In Fig. 1 (d), we have plotted the profile of $Re\left(\chi^{(3)}\right)$ as a function of probe detuning Δ_p/γ_{21} at resonant control field ($i.e.\, \Delta_c/\gamma_{21} = 0$), tuning the values of Ω_c/γ_{21} from 0 to 8. The plot of $Re\left(\chi^{(3)}\right)$ depicts that within the TW we have large Kerr nonlinear susceptibility of the order $\sim 10^{-12}\, m^2/V^2$. In order to study the effect of the control field detuning Δ_c/γ_{21} on Kerr nonlinearity, we have depicted profile of $Re\left(\chi^{(3)}\right)$ as a function of Δ_p/γ_{21} for different control field detunings ($\Delta_c/\gamma_{21} = -5\ to\ 5$), as shown in Fig. 1(e). From Fig. 1(e), it is evident that, the symmetry of the profile of $Re\left(\chi^{(3)}\right)$ is revered when the value of Δ_c/γ_{31} changes from positive to negative or vice versa and its value is still very large (of the order $\sim 10^{-12}\, \mathrm{m}^2/\mathrm{V}^2$). Thus, it is clear that within the EIT window, the exhibited Kerr nonlinearity is very large which could be tuned to any desired probe frequency by tuning the Rabi frequency and detuning of the control field.

Since it is clear from the Fig. 1(c), that the $Re\left(\chi^{(3)}\right)$ curves display a very steep positive slope at the centre of the EIT transparency window, therefore the group velocity of the probe laser can be slowed down with negligible absorption. Fig. 1(f) demonstrates the dependency of group velocity of the probe pulse on control field Ω_c/γ_{21}. We have depicted the variation of V_g/c with Δ_p/γ_{21} for different values of Ω_c/γ_{21}. From the Fig. 1(f) it is evident that with the enhancement of Ω_c/γ_{21}, initially V_g/c decreases but it poses a hump at and near the central probe frequency, thereafter it increases again. At the positions, $\Delta_p/\gamma_{21} = \pm 8$ for corresponding value of $\Omega_c/\gamma_{21} = 8$, the group velocity of the probe laser pulse is reduced by 10^4 times as compared to the velocity of light in vacuum.

Conclusion

By solving the steady-state Schrödinger equation analytically, we have demonstrated the linear and nonlinear susceptibilities for a weak probe and a coupling field in an asymmetric MQW nanostructure with a ladder-type configuration. Our system has achieved Kerr nonlinear susceptibilities of the order $\sim 10^{-12}\,\mathrm{m^2/V^2}$ at low absorption level facilitate by EIT. The group velocity of the probe laser pulse has been found to be slowed down by 10^4 times than the velocity of light in vacuum.

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References

[1] Harris, S. E. *Phys. Today* **1997**, *50*, 36.
[2] Chen, A. *Opt. Exp.* **2011**, *19*, 11944.
[3] Borgohain, N.; Konar, S. *J. Appl. Phys.* **2016**, *119*, 213103.
[4] Kivshar, Y. S.; Agarwal, G. P. *Optical Solitons: From fibers to photonic crystals*, **2003,** Academic Press New York.
[5] Zhu, C.; Huang, G. *Phys. Rev. B* **2009**, *80*, 235408.
[6] Swetanshumala, S.; Konar, S.; Biswas, A. *Appl. Phys. B* **2013**,*111*, 53.
[7] Faist, J.; Sirtori, C.; Capasso, F.; Chu, S. N.; Pfeiffer, L. N.; West, K. W. *Opt. Lett.* **1996**, *21*, 985.
[8] Silestri, L.; Bassani, F.; Czajkowski, G.; Davoudi, B. *Eur. Phys. J. B* **2002**, *27*, 89.
[9] Yang, W. X.; Zha, T. T.; Lee, R. K. *Phys. Lett. A* **2009**, *374*, 355.
[10] Frogley, M. D.; Dynes, J. F.; Beck, M.; Faist J.; Phillips, C. C. *Nature Materials* **2006**, *5*, 175.